Alg. 2 Warm Up #3-3

1. Solve by completing the square:

$x^2 + 8x - 3 = 0$

2. Solve:

a) $4^{3x} = 4^{5x - 8}$

b) $|3x - 2| > 5$

MATH NOTES

METHODS AND MEANINGS

p. 219

Notation for Inverses

When given a function $f(x)$, the notation for the inverse of the function is $f^{-1}(x)$. Note that the -1 is not a negative exponent. It is the mathematical symbol that indicates the "undo" or inverse function of $f(x)$.

For example, if $f(x) = x^3 - 1$ then $f^{-1}(x) = \sqrt[3]{x+1}$.

This same inverse notation is used to identify the inverse of trigonometric functions. For example the inverse of $\sin(x)$ is written $\sin^{-1}(x)$.

Graphs of inverses are a reflection in the line $y = x$

Inverse writing shortcut:

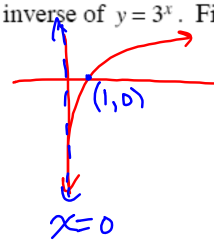
1) Swap x & y

2) Solve for y

HW Questions:

Review & Preview

5-60. In problem 5-56, you looked at the inverse of $y = 3^x$. Finish investigating this function.



$x = 3^y$

y= the exponent on 3 that gives us x.

5-61. Consider the function $f(x) = \frac{2}{7-x}$.

a. What is $f(7)$?

b. What is the domain of $f(x)$?

c. If $g(x) = 2x + 5$, what is $g(3)$?

d. Now use the output of $g(3)$ as the input for f to calculate $f(g(3))$.

#60) x-int: (1,0)
vertical asymptote at x=0
dom: $x > 0$
range: $y = \mathbb{R}$

5-62. Amanda wants to showcase her favorite function: $f(x) = 1 + \sqrt{x+5}$. She has built a function machine that performs these operations on the input values. Her brother Eric is always trying to mess up Amanda's stuff, so he created the inverse of $f(x)$, called it $e(x)$, and programmed it into a machine.

a. What is Eric's equation for his function $e(x)$?

b. What happens if the two machines are pushed together? What is $e(f(-4))$? Explain why this happens.

c. If $f(x)$ and $e(x)$ are graphed on the same set of axes, what would be true about the two graphs?

d. Draw the two graphs on the same set of axes. Be sure to show clearly the restricted domain and range of Amanda's function.

f(x)	e(x)
input x	input x
+5	−1
$\sqrt{\ }$	$(\ )^2$
+1	−5

a) $e(x) = (x-1)^2 - 5$

b)

5-63. Sketch the graph of $y+3=2^x$. → $y=2^x-3$

a. What are the domain and range of this function? d: $x=\mathbb{R}$ r: $y>-3$

b. Does this function have a line of symmetry? If so, what is it? No

c. What are the x- and y-intercepts?

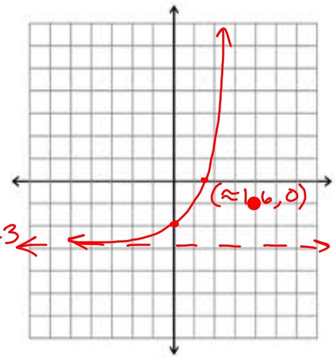
d. Change the equation so that the graph of the new equation has no x-intercepts.

Either reflect over the x-axis first:

$y=-2^x-3$

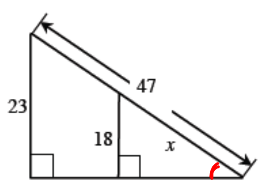
or move it up:

$y=2^x$

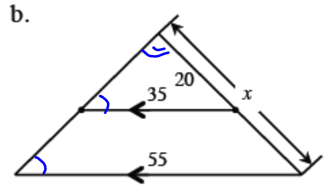


5-64. Solve for x in the following problems.

a.



b.



$$\frac{x}{47}=\frac{18}{23}$$

$$\frac{x}{20}=\frac{55}{35}$$

5-65. A woman plans to invest x dollars. Her investment counselor advises her that a safe plan is to invest 30% of that money in bonds and 70% in low risk stocks. The bonds currently have a simple interest rate of 7% and the stock has a dividend rate (like simple interest) of 9%.

a. Write an expression for the annual income that will come from the bond investment.

Let B = amount of $ earned in Bonds → $B = (0.30x)(0.07)$

$B = 0.021x$

b. Write an expression for the annual income that will come from the stock investment.

S = amount of $ earned in Stocks → $S = (0.70x)(0.09)$

$S = 0.063x$

c. Write an equation and solve it to find out how much the client needs to invest to have an annual income of $5,000.

$B + S = 5{,}000$

$0.021x + 0.063x = 5{,}000$

5-66. Factor each expression completely.

a. $x^2 - 49$ b. $6x^2 + 48x$ c. $x^2 - x - 72$ d. $2x^3 - 8x$

$2x(x^2 - 4)$

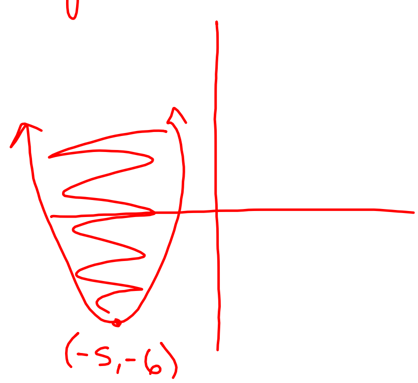
$2x(x+2)(x-2)$

5-67. Sketch the solution to this system of inequalities.

parabola left 5, down 6 No stretch or compression: → $y \geq (x+5)^2 - 6$

$y \leq -(x+4)^2 - 1$

reflect over x-axis left 4, down 1



Friday's Pink CP's:

5-56. THE INVERSE EXPONENTIAL FUNCTION

There are two parent functions, $y=|x|$ and $y=b^x$, that have inverses that you do not yet know how to write in y = form. You will come back to $y=|x|$ later. Since exponential functions are so useful for modeling situations in the world, the inverse of an exponential function is also important. Use $y=3^x$ as an example. Even though you may not know how to write the inverse of $y=3^x$ in y = form, you already know a lot about it.

a. You know how to make an $x \rightarrow y$ table for the inverse of $y=3^x$. Make the table.

b. You also know what the graph of the inverse looks like. Sketch the graph.

c. You also have one way to write the equation based on your algebraic shortcut that you used in part (d) of problem 5-40. Write an equation for the inverse, even though it may not be in y = form. $\boxed{x=3^y}$

d. If the input for the inverse function is 81, what is the output? If you could write an equation for this function in y = form, or as a function $g(x)=$, and you put in any number for x, how would you describe the outcome?

$81=3^y$

$g(x)=$ an exponent on the base 3 that gives me x.

5-57. AN ANCIENT PUZZLE

Parts (a) through (f) below are similar to a puzzle that is more than 2100 years old. Mathematicians first created the puzzle in ancient India in the 2nd century BC. More recently, about 700 years ago, Muslim mathematicians created the first tables allowing them to find answers to this type of puzzle quickly. Tables similar to them appeared in school math books until recently.

Here are some clues to help you figure out how the puzzle works:

$2^3=8$ $\quad \log_2 8 = 3$ ← exponent, base $\quad \boxed{\log_3 27 = 3} \rightarrow 3^3=27$

$5^2=25 \leftarrow \log_5 25 = 2 \quad \log_{10} 10{,}000 = 4 \rightarrow 10^4=10{,}000$

Use the clues to find the missing pieces of the puzzles below:

a. $\log_2 16 = ?$ 4 $\quad$ b. $\log_2 32 = ?$ 5 $\quad$ c. $\log_? 100 = 2$ 10

d. $\log_5 ? = 3$ $\quad$ e. $\log_? 81 = 4$ $\quad$ f. $\log_{100} 10 = ?$

$2^?=16$

$5^3=125$

$(?)^2=100$

$(?)^4=81$

$(100)^?=10$

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5-58. How is the Ancient Puzzle related to the problem of the inverse function for $y = 3^x$ in problem 5-56? Show how you can use the idea in the Ancient Puzzle to write an equation in $y =$ form or as $g(x) =$ for the inverse function in problem 5-56.

$x = 3^y$ → y = exponent on base 3 that $= x$

same

$$\boxed{y = \log_3 x}$$

CP's: 5- #68 ---> 70 (blue revised)

5.2.2 What is a logarithm?

Defining the Inverse of an Exponential Function

You have learned how to "undo" many different functions. However, the exponential function has posed some difficulty. In this lesson, you will learn more about the inverse exponential function. In particular, you will learn how to write an inverse exponential function in $y =$ form.

5-68. SILENT BOARD GAME

Your teacher will put an $x \rightarrow y$ table on the board or overhead that the whole class will work together to complete. The table will be like the one below. See which values you can fill in.

x	8	32	$\frac{1}{2}$	1	16	4	3	64	2	0	0.25 ($\frac{1}{4}$)	–1	$\sqrt{2}$	0.2	$\frac{1}{8}$
$g(x)$	3	5	–1	0	4	2	≈	6	1	[illegible]	-2	[illegible]	$\frac{1}{2}$		-3

exp. on 2

a. Describe an equation that relates x and $g(x)$.

b. Look back at the Ancient Puzzle in problem 5-57. If you have not already done so, use the idea of the Ancient Puzzle to write an equation for $g(x)$.

c. Why was it difficult to think of an output for the input of 0 or –1?

d. Find an output for $x = 25$ to the nearest hundredth.

CP's: 5- #68 ---> 70

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Base 2

x	8	32	$\frac{1}{2}$	1	16	4	3	64	2	0	0.25 ($\frac{1}{4}$)	–1	$\sqrt{2}$	0.2	$\frac{1}{8}$
$g(x)$ exponent	3	5	–1	0	4	2	≈	6	1	∅	-2	∅	$\frac{1}{2}$	≈	-3

a. Describe an equation that relates x and $g(x)$. = an exponent on base 2 that = x.

$2^{g(x)} = x$

b. Look back at the Ancient Puzzle in problem 5-57. If you have not already done so, use the idea of the Ancient Puzzle to $\boxed{\text{write an equation for } g(x)}$.

$g(x) = \log_2 x$

c. Why was it difficult to think of an output for the input of 0 or –1?

d. $\boxed{\text{Find an output for } x = 25}$ to the nearest hundredth. → $g(25) \approx 4.64$

$2^3 = 8$ $\quad 2^0 = 1$

$2^{-1} = \frac{1}{2}$ $\quad 2^5 = 32$

$2^6 = 64$ $\quad 2^? \neq 0$

$2^{1/2} = \sqrt{2}$ $\quad 2^? \neq -1$

2^1	$2^?$	2^2
2	3	4

2^{-3}	$2^?$	2^{-2}
$\frac{1}{8}$	0.2 ($\frac{1}{5}$)	$\frac{1}{4}$

2^4	$2^?$	2^5
16	25	32

$2^{4.65} \approx 25.11$

$2^{4.64} \approx 24.93$

5-69. ANOTHER LOGARITHM TABLE

Lynn was supposed to fill in this table for $g(x) = \log_5 x$. She thought she could use the log button on her calculator, but when she tried to enter 5, 25, and 125, she did not get the outputs the table below displays. She was fuming over how long it was going to take to guess and check each one when her sister suggested that she did not have to do that for all of them. She could fill in a few more and then use what she knew about exponents to figure out some of the others.

use guess method

x	$\frac{1}{25}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	3	4	5	6	7	8	10	25	100	125	625
$g(x)$	-2	-1		0				1					2		3	

do last

a. Discuss with your team which outputs can be filled in without a calculator. Fill those in and explain how you found these entries.

b. With your team, use your calculator to estimate the remaining values of $g(x)$ to the nearest hundredth. Once you have entered several, use your knowledge of exponent rules to see if you can find any shortcuts.

c. What do you notice about the results for $g(x)$ as x increases?

d. Use your table to draw the graph of $y = \log_5 x$. How does your graph compare to the graph of $y = 5^x$?

5-70. Find each of the values below, and then justify your answers by writing the equivalent exponential form.

a. $\log_2(32) = ?$ b. $\log_2(\frac{1}{2}) = ?$ c. $\log_2(4) = ?$ d. $\log_2(0) = ?$

e. $\log_2(?) = 3$ f. $\log_2(?) = \frac{1}{2}$ g. $\log_2(\frac{1}{16}) = ?$ h. $\log_2(?) = 0$

HW: 5 -

73 ---> 80

Friday's Short Quiz:

* Write an inverse equation.

* Graph an inverse, state domain & range.

* Solve a multi step absolute value inequality.